# Chapter 3: Outline Solutions

The macro file ES Macro\_Chapter 3 contains all the data sets used in Exercises for Chapter 3. The macro is being updated so the layout may change but the results should not. Users are encouraged to run the macro to gain insight into methods’ performance. ONLY out-of-sample results are recorded here, in most cases.

### 3.1 Excel solution in Chapter\_3\_solutions.xlsx

(a) The spreadsheet for the calculations is shown below. The error measures are calculated over the period 2001-2015, but any other reasonable period could be used.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Year | 3-term | 7-term | Actual | Abs\_error\_3 | Abs\_error\_7 | Sq\_error\_3 | Eq\_error\_7 |
| 2001 | 6.13 | 5.94 | 3.3 | 2.83 | 2.64 | 8.03 | 6.98 |
| 2002 | 5.37 | 5.51 | 3.3 | 2.07 | 2.21 | 4.27 | 4.90 |
| 2003 | 4.37 | 5.29 | 4.9 | 0.53 | 0.39 | 0.28 | 0.15 |
| 2004 | 3.83 | 5.17 | 6.6 | 2.77 | 1.43 | 7.65 | 2.04 |
| 2005 | 4.93 | 5.21 | 6.7 | 1.77 | 1.49 | 3.12 | 2.21 |
| 2006 | 6.07 | 5.37 | 5.8 | 0.27 | 0.43 | 0.07 | 0.18 |
| 2007 | 6.37 | 5.30 | 4.5 | 1.87 | 0.80 | 3.48 | 0.64 |
| 2008 | 5.67 | 5.01 | 1.7 | 3.97 | 3.31 | 15.73 | 10.98 |
| 2009 | 4.00 | 4.79 | -2.0 | 6.00 | 6.79 | 36.00 | 46.05 |
| 2010 | 1.40 | 4.03 | 3.8 | 2.40 | 0.23 | 5.76 | 0.05 |
| 2011 | 1.17 | 3.87 | 3.7 | 2.53 | 0.17 | 6.42 | 0.03 |
| 2012 | 1.83 | 3.46 | 4.1 | 2.27 | 0.64 | 5.14 | 0.41 |
| 2013 | 3.87 | 3.09 | 3.3 | 0.57 | 0.21 | 0.32 | 0.05 |
| 2014 | 3.70 | 2.73 | 4.2 | 0.50 | 1.47 | 0.25 | 2.17 |
| 2015 | 3.87 | 2.69 | 3.7 | 0.17 | 1.01 | 0.03 | 1.03 |
|  |  |  | Mean | 2.03 | 1.55 | 6.44 | 5.19 |
|  |  |  |  |  | RMSE | 2.54 | 2.28 |
|  |  |  |  |  |  |  |  |

 (b) The MA7 forecasts show somewhat lower values for the MAE and for the MSE. The MAPE is not appropriate because the observations may be negative.

### 3.2 Excel solution in Chapter\_3\_solutions.xlsx

(a) The spreadsheet for the calculations is shown below. The error measures are calculated over the period 2001-2015, but any other reasonable period could be used.1998 - 2007.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Year | 3-term | 7-term | Actual | Abs\_error\_3 | Abs\_error\_7 | Sq\_error\_3 | Eq\_error\_7 |
| 2001 | 2.37 | 2.54 | 2.8 | 0.45 | 0.28 | 0.21 | 0.08 |
| 2002 | 2.80 | 2.57 | 1.6 | 1.21 | 0.99 | 1.47 | 0.98 |
| 2003 | 2.60 | 2.40 | 2.3 | 0.33 | 0.13 | 0.11 | 0.02 |
| 2004 | 2.23 | 2.31 | 2.7 | 0.45 | 0.37 | 0.20 | 0.14 |
| 2005 | 2.18 | 2.35 | 3.4 | 1.21 | 1.04 | 1.48 | 1.08 |
| 2006 | 2.78 | 2.62 | 3.2 | 0.45 | 0.61 | 0.20 | 0.37 |
| 2007 | 3.10 | 2.77 | 2.9 | 0.25 | 0.09 | 0.06 | 0.01 |
| 2008 | 3.16 | 2.69 | 3.8 | 0.68 | 1.15 | 0.47 | 1.32 |
| 2009 | 3.31 | 2.83 | -0.4 | 3.66 | 3.19 | 13.41 | 10.18 |
| 2010 | 2.11 | 2.56 | 1.6 | 0.47 | 0.92 | 0.22 | 0.84 |
| 2011 | 1.71 | 2.47 | 3.2 | 1.45 | 0.69 | 2.10 | 0.48 |
| 2012 | 1.48 | 2.54 | 2.1 | 0.59 | 0.47 | 0.35 | 0.22 |
| 2013 | 2.29 | 2.35 | 1.5 | 0.82 | 0.88 | 0.68 | 0.78 |
| 2014 | 2.23 | 2.10 | 1.6 | 0.61 | 0.47 | 0.37 | 0.22 |
| 2015 | 1.72 | 1.92 | 0.1 | 1.60 | 1.80 | 2.56 | 3.24 |
|  |  |  | Mean | 0.95 | 0.87 | 1.59 | 1.33 |
|  |  |  |  |  | RMSE | 1.26 | 1.15 |
|  |  |  |  |  |  |  |  |

(b) The MA7 forecasts show somewhat lower values for the MAE and for the MSE. The MAPE is not appropriate because the observations may be negative.

### 3.3

a) Results are given for out-of-sample using the ESM.

α=0.2

|  |
| --- |
| **Forecasting Information for Single Exponential smoothing** |
|   |   |
| Total Observations | 53 |
| Holdout | 15 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.7468 |
| Mean of Squared Error (MSE) | 4.8806 |
| Root Mean of Squared Error (RMSE) | 2.2092 |
| Mean of Absolute Percent Error (MAPE) | 0.2582 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.2264 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.4777 |
| Mean of Squared Error (MSE) | 5.2127 |
| Root Mean of Squared Error (RMSE) | 2.2831 |
| Mean of Absolute Percent Error (MAPE) | 0.1154 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.5691 |
|   |   |
| **Parameter** |   |
| alpha | 0.2000 |
|   |   |

α=0.5

|  |
| --- |
| **Forecasting Information for Single Exponential smoothing** |
|   |   |
| Total Observations | 53 |
| Holdout | 15 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.6768 |
| Mean of Squared Error (MSE) | 4.4987 |
| Root Mean of Squared Error (RMSE) | 2.1210 |
| Mean of Absolute Percent Error (MAPE) | 0.2511 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.2184 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.7413 |
| Mean of Squared Error (MSE) | 5.0368 |
| Root Mean of Squared Error (RMSE) | 2.2443 |
| Mean of Absolute Percent Error (MAPE) | 0.2332 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.8681 |
| **Parameter** |   |
| alpha | 0.5000 |
|   |   |

α=0.8

|  |
| --- |
| **Forecasting Information for Single Exponential smoothing** |
|   |   |
| Total Observations | 53 |
| Holdout | 15 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.8363 |
| Mean of Squared Error (MSE) | 5.3158 |
| Root Mean of Squared Error (RMSE) | 2.3056 |
| Mean of Absolute Percent Error (MAPE) | 0.2715 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.2404 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.7066 |
| Mean of Squared Error (MSE) | 4.9516 |
| Root Mean of Squared Error (RMSE) | 2.2252 |
| Mean of Absolute Percent Error (MAPE) | 0.2735 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 2.3133 |
|   |   |
| **Parameter** |   |
| alpha | 0.8000 |
|   |   |

The out-of-sample summary results are shown in the table.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Criterion | α =0.2 | α=0.5 | α=0.8 | MA(3) | MA(7) |
| MAE | 1.48 | 1.74 | 1.71 | 2.03 | 1.55 |
| RMSE | 2.28 | 2.24 | 2.23 | 2.54 | 2.28 |

(b) No clear preference. Note that MAPE not useful in this example.

(c) MA(7) and SES(0.2) perform very similarly. MA(3) is inferior.

### 3.4

The ESM gives the following out-of-sample results [Recall that best fit in-sample does not imply the best performance in the hold-out sample.]

|  |
| --- |
| **Forecasting Information for Single Exponential smoothing** |
|   |   |
| Total Observations | 53 |
| Holdout | 15 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.6310 |
| Mean of Squared Error (MSE) | 4.3857 |
| Root Mean of Squared Error (RMSE) | 2.0942 |
| Mean of Absolute Percent Error (MAPE) | 0.2448 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.2121 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.6578 |
| Mean of Squared Error (MSE) | 5.0538 |
| Root Mean of Squared Error (RMSE) | 2.2481 |
| Mean of Absolute Percent Error (MAPE) | 0.1907 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.7211 |
|   |   |
| **Parameter** |   |
| alpha | 0.3768 |
|  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| Method | MA(3) | MA(7) | SES (α=0.377) |
| MAE | 2.03 | 1.55 | 1.65 |
| RMSE | 2.54 | 2.28 | 2.25 |

The SES and MA(7) methods appears to perform equally well.

### 3.5

a) Results are given for out-of-sample using the ESM.

α=0.2

|  |
| --- |
| **Forecasting Information for Single Exponential smoothing** |
|   |   |
| Total Observations | 53 |
| Holdout | 15 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.9922 |
| Mean of Squared Error (MSE) | 6.8099 |
| Root Mean of Squared Error (RMSE) | 2.6096 |
| Mean of Absolute Percent Error (MAPE) | 0.4670 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.4178 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 0.8596 |
| Mean of Squared Error (MSE) | 1.4373 |
| Root Mean of Squared Error (RMSE) | 1.1989 |
| Mean of Absolute Percent Error (MAPE) | 0.6865 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.4968 |
|   |   |
| **Parameter** |   |
| alpha | 0.2000 |
|   |   |

α=0.5

|  |  |
| --- | --- |
| Total Observations | 53 |
| Holdout | 15 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.5932 |
| Mean of Squared Error (MSE) | 4.6471 |
| Root Mean of Squared Error (RMSE) | 2.1557 |
| Mean of Absolute Percent Error (MAPE) | 0.3544 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.3233 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 0.8320 |
| Mean of Squared Error (MSE) | 1.5584 |
| Root Mean of Squared Error (RMSE) | 1.2484 |
| Mean of Absolute Percent Error (MAPE) | 0.4043 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.4778 |
|   |   |
| **Parameter** |   |
| alpha | 0.5000 |
|   |   |

α=0.8

|  |  |
| --- | --- |
|   |   |
| Total Observations | 53 |
| Holdout | 15 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.4407 |
| Mean of Squared Error (MSE) | 3.8691 |
| Root Mean of Squared Error (RMSE) | 1.9670 |
| Mean of Absolute Percent Error (MAPE) | 0.3097 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.2924 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 0.9715 |
| Mean of Squared Error (MSE) | 1.8532 |
| Root Mean of Squared Error (RMSE) | 1.3613 |
| Mean of Absolute Percent Error (MAPE) | 0.3626 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.5694 |
|   |   |
| **Parameter** |   |
| alpha | 0.8000 |
|   |   |
|   |   |

Statistics apply to the hold-out sample

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Criterion | α =0.2 | α=0.5 | α=0.8 | MA(3) | MA(7) |
| MAE | 0.86 | 0.83 | 0.97 | 0.95 | 0.87 |
| RMSE | 1.20 | 1.25 | 1.36 | 1.26 | 1.15 |

In this case the MA(7) results are best on RMSE, but α =0.5 on MAE.

### 3.6

|  |
| --- |
| **Forecasting Information for Single Exponential smoothing** |
|   |   |
| Total Observations | 53 |
| Holdout | 15 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.4113 |
| Mean of Squared Error (MSE) | 3.4422 |
| Root Mean of Squared Error (RMSE) | 1.8553 |
| Mean of Absolute Percent Error (MAPE) | 0.2990 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.2901 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.0789 |
| Mean of Squared Error (MSE) | 2.1226 |
| Root Mean of Squared Error (RMSE) | 1.4569 |
| Mean of Absolute Percent Error (MAPE) | 0.3794 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.7132 |
|   |   |
| **Parameter** |   |
| alpha | 1.0000 |
|   |   |
|   |   |

The SES macro gives the optimal value on the boundary as α = 1.00. This value leads to inferior out-of-sample performance. The outlier associated with the recession is the main reason for this anomalous result.

### 3.7 and 3.8

a) α =0.2

|  |
| --- |
| **Forecasting Information for Single Exponential smoothing** |
|   |   |
| Total Observations | 38 |
| Holdout | 8 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 8.6157 |
| Mean of Squared Error (MSE) | 152.0103 |
| Root Mean of Squared Error (RMSE) | 12.3292 |
| Mean of Absolute Percent Error (MAPE) | 0.2931 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.3124 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 30.7436 |
| Mean of Squared Error (MSE) | 1150.3005 |
| Root Mean of Squared Error (RMSE) | 33.9161 |
| Mean of Absolute Percent Error (MAPE) | 0.3606 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.4006 |
|   |   |
| **Parameter** |   |
| alpha | 0.2000 |
|   |   |
|   |   |

α =0.5

|  |
| --- |
| **Forecasting Information for Single Exponential smoothing** |
|   |   |
| Total Observations | 38 |
| Holdout | 8 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 6.3615 |
| Mean of Squared Error (MSE) | 77.2537 |
| Root Mean of Squared Error (RMSE) | 8.7894 |
| Mean of Absolute Percent Error (MAPE) | 0.2340 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.2357 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 20.6603 |
| Mean of Squared Error (MSE) | 664.6315 |
| Root Mean of Squared Error (RMSE) | 25.7804 |
| Mean of Absolute Percent Error (MAPE) | 0.2793 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.2541 |
|   |   |
| **Parameter** |   |
| alpha | 0.5000 |
|   |   |
|   |   |

α =0.8

|  |  |
| --- | --- |
|   |   |
| Total Observations | 38 |
| Holdout | 8 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 5.5590 |
| Mean of Squared Error (MSE) | 49.2875 |
| Root Mean of Squared Error (RMSE) | 7.0205 |
| Mean of Absolute Percent Error (MAPE) | 0.2197 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.2188 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 20.1229 |
| Mean of Squared Error (MSE) | 616.4563 |
| Root Mean of Squared Error (RMSE) | 24.8285 |
| Mean of Absolute Percent Error (MAPE) | 0.2824 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.2467 |
|   |   |
| **Parameter** |   |
| alpha | 0.8000 |
|   |   |
|   |   |

b)

optimal α

|  |
| --- |
| **Forecasting Information for Single Exponential smoothing** |
|   |   |
| Total Observations | 38 |
| Holdout | 8 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 5.2641 |
| Mean of Squared Error (MSE) | 40.9550 |
| Root Mean of Squared Error (RMSE) | 6.3996 |
| Mean of Absolute Percent Error (MAPE) | 0.2182 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.2171 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 19.9750 |
| Mean of Squared Error (MSE) | 615.5423 |
| Root Mean of Squared Error (RMSE) | 24.8101 |
| Mean of Absolute Percent Error (MAPE) | 0.2849 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.2473 |
|   |   |
| **Parameter** |   |
| alpha | 1.0000 |
|   |   |
|   |   |

c/d Statistics apply to the hold-out sample

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Criterion | α =0.2 | α=0.5 | α=0.8 | SES | LES |
| MAE | 30.7 | 20.7 | 20.1 | 20.0 | 23.7 |
| RMSE | 33.9 | 25.8 | 24.8 | 24.8 | 27.3 |
| MAPE | 36.1 | 27.9 | 28.2 | 28.5 | 33.0 |
| MdAE | 30.6 | 15.8 | 18.4 | 19.7 | 16.9 |
| MdAPE | 32.7 | 19.4 | 20.6 | 22.4 | 19.1 |

Forecasts available via the ESM. The summary statistics appear in the table. Occasional large errors have a considerable effect on the ordering of the methods. These large errors mean that none of the different methods perform well.

### 3.9

Statistics apply to the hold-out sample

The summary statistics are:

|  |  |  |  |
| --- | --- | --- | --- |
| Statistic | 1978-2007 | 1978-2004 | 1989-2004 |
| MAE | 23.7 | 18.2 | 19.3 |
| RMSE | 27.3 | 23.3 | 25.1 |
| MAPE | 33.0 | 26.0 | 29.9 |
| α | 0.96 | 0.95 | 0.23 |
| β | 0.43 | 0.36 | 1.00 |
| SE | 6.11 | 6.11 | 4.30 |
| 95% PIF+/- value | 11.98 | 11.98 | 8.43 |
| Inside PI? | 1/8 | 1/8 | 1/8 |

Omitting the start of the series changes the estimates considerably.

If the absolute error is less than the PI figure in the table, the PI includes the observation; otherwise not.

The end of the series is much more volatile than the start, so none of the PIs comes close to including sufficient observations.

### 3.10

Statistics apply to the hold-out sample

|  |  |  |
| --- | --- | --- |
|  | **SES** | **LES** |
| Statistic | 2000-2011 | 2002-2011 | 2004-2011 | 2000-2011 | 2002-2011 | 2004-2011 |
| MAE | 73.2 | 73.2 | 73.2 | 15.2 | 15.2 | 15.2 |
| RMSE | 75.1 | 75.1 | 75.1 | 21.7 | 21.7 | 21.7 |
| MAPE | 5.3 | 5.3 | 5.3 | 1.2 | 1.2 | 1.2 |
| α | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| β |  |  |  | 0.39 | 0.39 | 0.39 |
| 95% PIF+/- value | 42.4 | 44.2 | 47.5 | 37.4 | 41.4 | 45.8 |
| Inside PI? | 5/16 | 5/16 | 6/16 | 13/16 | 13/16 | 14/16 |

Data fitted for three periods, all ending in 2011Q4. Two complete years of data were removed each time.

The SES totally fails to keep up with sales growth and the performance is poor; LES does much better. Note that the fits are identical because α = 1 but the PIs widen when the early observations are removed because those observations are so small. The LES provides much better coverage even though the PIs are narrower.

### 3.11/3.12/3.15/3.16

1. Statistics apply to the hold-out sample

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Statistic | 1963-2003 | 1963-2010 | 1986-2003 | 1986-2010 | 1963-2003\_Log |
| Holdout | 12 | 5 | 12 | 5 | 12 |
| MAE | 1666 | 1706 | 1697 | 617 | 1829 |
| RMSE | 2362 | 2385 | 2474 | 680 | 2588 |
| MAPE | 9.2 | 9.5 | 9.4 | 4.0 | 10.3 |
| α | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| β | 0.0014 | 0.00 | 0.12 | 0.047 | 0.098 |
| 95% PIF+/- value | 2089 | 2458 | 2683 | 4136 | 0.312\* |
| Inside PI? | 9/12 | 5/5 | 9/12 | 5/5 | 11/12 |

1. The log interval, after transformation, produces multipliers of 0.732 and 1.366.
2. Eliminating the first part of the series has little impact on the parameter estimates or on the (point) forecast performance However the PIs are wider because the variance is estimated using errors that are larger in absolute value in the later part of the series.
3. The forecasts for 2004-2006 were wide of the mark, which explains the differences in coverage. The log coverage is better because the intervals are wider.
4. The damped trend method reduces to to LES (phi = 1), as does the Box-Cox transform.
5. The resulting estimates are close to the random walk, so the *RelMAE* values are very close to 1.0.

### 3.13/3.14

|  |  |  |  |
| --- | --- | --- | --- |
| Statistic | LES | DES | LES+Log |
| MAE | 23.7 | 22.6 | 25.3 |
| RMSE | 27.3 | 26.1 | 29.7 |
| MAPE | 33.0 | 31.2 | 35.1 |
| α | 0.96 | 0.79 | 1.00 |
| β | 0.43 | 0.89 | 0.45 |
| φ |  | 0.64 |  |
| SE | 6.11 | 5.79 | 0.300\* |
| 95% PIF+/- value | 11.98 | 11.35 | 0.594\* |
| Inside PI? | 1/8 | 2/8 | 6/8 |

\* Natural log scale

The ESM calculates the MAE etc. after converting back to the original units.

The PI is calculated for the log values and then transformed back so the upper limit if Forecast\*exp(0.594) yielding a multiplier of 1.81, which produces a very wide interval. Hence the ‘improved’ coverage.

### 3.17

 Using the estimation sample 1963-2003 there are 38 estimated errors. The three largest negative errors are -2006, -1479 and -1054. The three largest positive errors are 1600, 3460 and 3714. Taking the second value in each case yields a 92% interval, whereas taking the first three yields an 87% interval. What we decide to do does not matter much at the positive end but clearly makes a difference at the lower end. For simplicity, we take the average of the second and third errors in each tail so that the prediction interval becomes . Clearly the interval is not symmetric about the point forecast. Intervals based upon the normal distribution would be . Although the normal intervals are narrower, their symmetric nature in this case captures 9 out of 12 forecasts whereas the asymmetric intervals capture only 8 out of 12.



3.18 Write  It follows that 

 The log series expansion is 

 When *a* is small we have that 

### 3.19

The trend equation is .

 The level equation may be written as: 

Thus, the trend equation becomes: 

### 3.20

From (3.13), we have:



At time *t*, we do not know the value of the error term for time *(t+1),* so we replace it by a “best guess” of zero (This step is justified in Chapter 5). Thus, when we forecast two steps ahead we have:



Repeating this step leads to equation (3.14).

### 3.21

|  |
| --- |
| **Forecasting Information for Additive Trend Exponential smoothing** |
|   |   |
| Total Observations | 38 |
| Holdout | 8 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 4.9947 |
| Mean of Squared Error (MSE) | 35.9399 |
| Root Mean of Squared Error (RMSE) | 5.9950 |
| Mean of Absolute Percent Error (MAPE) | 0.2285 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.2303 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 23.8911 |
| Mean of Squared Error (MSE) | 759.0509 |
| Root Mean of Squared Error (RMSE) | 27.5509 |
| Mean of Absolute Percent Error (MAPE) | 0.3327 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.2782 |
|   |   |
| **Parameter** |   |
| alpha | 0.9678 |
| beta | 0.4434 |
| lambda | 0.8796 |
|   |   |
|   |   |

|  |  |  |
| --- | --- | --- |
| Statistic | LES | LES (Box-Cox) |
| MAE | 23.7 | 23.9 |
| RMSE | 27.3 | 27.6 |
| MAPE | 33.0 | 33.3 |
| α | 0.96 | 0.97 |
| β | 0.43 | 0.44 |
| φ |  | 0.88 |
| SE | 6.11 | 6.10 |
| 95% PIF+/- value | 11.98 | 11.97 |
| Inside PI? | 1/8 | 1/8 |

The Box-Cox transform does not offer any improvement in this case.

### Minicase 3.1

The data show cyclical effects but not systematic trend. Check SES, LES and DES.

### Minicase 3.2

Forecast developments for the three types of store (clear trends). Possible correlations because of store upgrades?

### Minicase 3.3

Returns will come out very close to a random walk, as expected. Is Volatility predictable? Check using SES and possibly DES.