# Chapter 4: Outline Solutions

The macro file ES Macro\_Chapter 4 contains all the data sets used in Exercises for Chapter 4. The macro is being updated so the layout may change but the results should not. Users are encouraged to run the macro to gain insight into methods’ performance. ONLY out-of-sample results are recorded here, in most cases.

Note that the output includes many more decimal points than should be used when presenting results. Excel should be used to format them for presentational purposes.

### 4.1

The plots show the increased level of sales in the fourth quarter and growth over time so that a seasonal method with trend will be required.





### 4.2

The forecasts are shown in the spreadsheet. Thus, for year 3, quarter 1 the forecast is:





The layout of the spreadsheet is shown below.



### 4.3

Using the ESM we obtain

Using data to 12/2013

|  |
| --- |
| **Forecasting Information for Additive Seasonal Exponential smoothing** |
|   |   |
| Total Observations | 180 |
| Holdout | 24 |
| Forecast Horizon | 10 |
| Seasonal Cycle | 12 |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.008367 |
| Mean of Squared Error (MSE) | 1.512719 |
| Root Mean of Squared Error (RMSE) | 1.229926 |
| Mean of Absolute Percent Error (MAPE) | 0.010070 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.010068 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 0.862303 |
| Mean of Squared Error (MSE) | 0.887526 |
| Root Mean of Squared Error (RMSE) | 0.942086 |
| Mean of Absolute Percent Error (MAPE) | 0.008668 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.008663 |
|   |   |
| **Parameter** |   |
| alpha | 0.000000 |
| gamma | 0.239981 |

Data to 12/2014

|  |
| --- |
| **Forecasting Information for Additive Seasonal Exponential smoothing** |
|   |   |
| Total Observations | 180 |
| Holdout | 12 |
| Forecast Horizon | 10 |
| Seasonal Cycle | 12 |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 0.999151 |
| Mean of Squared Error (MSE) | 1.472396 |
| Root Mean of Squared Error (RMSE) | 1.213423 |
| Mean of Absolute Percent Error (MAPE) | 0.009984 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.009982 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 0.785601 |
| Mean of Squared Error (MSE) | 0.762181 |
| Root Mean of Squared Error (RMSE) | 0.873030 |
| Mean of Absolute Percent Error (MAPE) | 0.007858 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.007851 |
|   |   |
| **Parameter** |   |
| alpha | 0.000000 |
| gamma | 0.223485 |
|   |   |

The parameter estimates are very similar; the error measures are slightly lower for 2014 only. Overall, the pattern is very stable as expected.

### 4.4

The output from the macro *Exponential Smoothing Macro (ESM)* is summarized below. The holdout period corresponds to the last two years (2014 and 2015). The table compares the out-of-sample summary statistics. The seasonals provide a small improvement: the Holt-Winters Additive method performs best but the Multiplicative method produces similar results.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Out of sample Error** |  |  |  |  |  |
| **Seasonal** | No | Add | Mult | Add | Mult |
| **Trend**  | No | No | No | Yes | Yes |
| Mean of Absolute Deviation (MAD) | 187.9 | 180.1 | 180.4 | 169.0 | 169.8 |
| Root Mean of Squared Error (RMSE) | 230.9 | 229.5 | 230.7 | 221.6 | 222.2 |
| Mean of Absolute Percent Error (MAPE) | 3.8 | 3.6 | 3.6 | 3.4 | 3.4 |
|   |  |  |  |  |  |
| **Parameter** |  |  |  |  |  |
| alpha | 0.67 | 0.55 | 0.56 | 0.51 | 0.55 |
| beta |  |  |  | 0.06 | 0.03 |
| gamma |  | 0.45 | 0.42 | 0.45 | 0.40 |

### 4.5

The Additive Holt-Winters method performs best according to RMSE, but the multiplicative version does better according to MAE and MAPE

|  |
| --- |
| **Forecasting Information for Additive Trend Additive Seasonal Exponential smoothing (with log transform)** |
|   |   |
| Total Observations | 180 |
| Holdout | 36 |
| Forecast Horizon | 10 |
| Seasonal Cycle | 12 |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 295.314207 |
| Mean of Squared Error (MSE) | 145916.442829 |
| Root Mean of Squared Error (RMSE) | 381.990108 |
| Mean of Absolute Percent Error (MAPE) | 0.034708 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.034584 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 378.022859 |
| Mean of Squared Error (MSE) | 189214.365079 |
| Root Mean of Squared Error (RMSE) | 434.987776 |
| Mean of Absolute Percent Error (MAPE) | 0.034493 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.034608 |
|   |   |
| **Parameter** |   |
| alpha | 0.090205 |
| beta | 0.000000 |
| gamma | 1.000000 |
|   |   |
|   |   |
| **Out of sample Error** |  |  |  |  |  |
| **Trend** | No | Yes | Yes | Yes-Log |  |
| **Seasonal** | Add | Add | Mult | Log-Add |  |
| Mean of Absolute Deviation (MAD) | 386.2 | 378.0 | 369.5 | 372.6 |  |
| Root Mean of Squared Error (RMSE) | 454.8 | 435.0 | 456.6 | 457.5 |  |
| Mean of Absolute Percent Error (MAPE) | 3.5 | 3.4 | 3.3 | 3.4 |  |
|   |   |   |   |   |  |
| **Parameter** |   |   |   |   |  |
| alpha | 0.214 | 0.090 | 0.096 | 0.123 |  |
| beta |  | 0.000 | 0.000 | 0.020 |  |
| gamma | 1.000 | 1.000 | 0.268 | 0.268 |  |

### 4.6

The transform to logs works best, but it is very similar to the Multiplicative Holt-Winters case.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Out of sample Error** |  |  |  |  |
| **Trend** | No | Yes | Yes | Yes-Log |
| **Seasonal** | Add | Add | Mult | Log-Add |
| Mean of Absolute Deviation (MAD) | 5.88 | 5.61 | 4.87 | 4.79 |
| Root Mean of Squared Error (RMSE) | 6.89 | 6.73 | 5.77 | 5.71 |
| Mean of Absolute Percent Error (MAPE) | 1.54 | 1.47 | 1.28 | 1.26 |
|   |   |   |   |   |
| **Parameter** |   |   |   |   |
| alpha | 0.65 | 0.62 | 0.66 | 0.66 |
| beta |  | 0.00 | 0.00 | 0.00 |
| gamma | 0.27 | 0.26 | 0.40 | 0.40 |

### 4.7

The model with no trend term and additive seasonal works best. This seems odd because the series clearly displays a marked trend. The reason for this result is that the seasonal component has a trend removal aspect to it. (see Chapter 5 on differences).

The analysis would not be very useful for other movies because Titanic had a much longer run in theaters than any other movie in recent history, so the results would not be comparable.

|  |
| --- |
| **Forecasting Information for Additive Seasonal Exponential smoothing** |
|   |   |
| Total Observations | 186 |
| Holdout | 28 |
| Forecast Horizon | 10 |
| Seasonal Cycle | 7 |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 282.959400 |
| Mean of Squared Error (MSE) | 182725.362800 |
| Root Mean of Squared Error (RMSE) | 427.463873 |
| Mean of Absolute Percent Error (MAPE) | 0.351907 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.416836 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 66.458418 |
| Mean of Squared Error (MSE) | 7761.442363 |
| Root Mean of Squared Error (RMSE) | 88.099049 |
| Mean of Absolute Percent Error (MAPE) | 0.314476 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.323620 |
|   |   |
| **Parameter** |   |
| alpha | 0.357710 |
| gamma | 0.527227 |
|   |   |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Out of sample Error** |  |  |  |  |
| **Trend** | No | Yes | Yes | Yes-Log |
| **Seasonal** | Add | Add | Mult | Log-Add |
| Mean of Absolute Deviation (MAD) | 66.5 | 71.3 | 92.3 | 76.3 |
| Root Mean of Squared Error (RMSE) | 88.1 | 93.7 | 132.1 | 111.3 |
| Mean of Absolute Percent Error (MAPE) | 31.4 | 33.8 | 36.5 | 29.6 |
|   |   |   |   |   |
| **Parameter** |   |   |   |   |
| alpha | 0.36 | 0.49 | 0.56 | 0.52 |
| beta |   | 0.04 | 0.02 | 0.02 |
| gamma | 0.53 | 0.50 | 0.56 | 0.58 |

### 4.8

The multiplicative Holt-Winters method produces the best results, although the Additive method with a log transform almost as well.

|  |
| --- |
| **Forecasting Information for Additive Trend Multiplicative Seasonal Exponential smoothing** |
|   |   |
| Total Observations | 192 |
| Holdout | 36 |
| Forecast Horizon | 10 |
| Seasonal Cycle | 12 |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 12.007355 |
| Mean of Squared Error (MSE) | 273.071694 |
| Root Mean of Squared Error (RMSE) | 16.524881 |
| Mean of Absolute Percent Error (MAPE) | 0.050778 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.050672 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 9.716187 |
| Mean of Squared Error (MSE) | 192.841887 |
| Root Mean of Squared Error (RMSE) | 13.886752 |
| Mean of Absolute Percent Error (MAPE) | 0.033668 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.032786 |
|   |   |
| **Parameter** |   |
| alpha | 1.000000 |
| beta | 0.005189 |
| gamma | 0.129685 |
|   |   |
|   |   |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Out of sample Error** |  |  |  |  |  |  |
| **Trend** | No | Yes | No | Yes | Yes | Yes-Log |
| **Seasonal** | No | No | Add | Add | Mult | Log-Add |
| Mean of Absolute Deviation (MAD) | 12.38 | 12.37 | 9.97 | 10.05 | 9.72 | 9.75 |
| Root Mean of Squared Error (RMSE) | 16.62 | 16.31 | 14.37 | 14.44 | 13.89 | 13.96 |
| Mean of Absolute Percent Error (MAPE) | 4.44 | 4.28 | 3.56 | 3.59 | 3.37 | 3.37 |
|   |   |   |   |   |   |   |
| **Parameter** |   |   |   |   |   |   |
| alpha | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| beta |  | 0.82 |  | 0.00 | 0.01 | 0.00 |
| gamma |   |   | 0.09 | 0.09 | 0.13 | 0.13 |

###  4.9

The example was designed to illustrate the additive seasonal pattern so the multiplicative form works less well. The commands for the first part of the spreadsheet are shown in the second table.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Obs | Series | MA(4) | Detrended Series | Seasonal Factor | Deseason-alized Series | Deseason-alized Forecasts | Series Forecasts | Error | Error Squared | Absolute Error | Absolute Percentage Error |
| 1 | 115 |  |  | 1.18 | 97.48 |  |  |  |  |  |  |
| 2 | 90 |  |  | 0.87 | 103.34 |  |  |  |  |  |  |
| 3 | 65 |  |  | 0.68 | 95.79 |  |  |  |  |  |  |
| 4 | 135 |  |  | 1.27 | 106.23 |  |  |  |  |  |  |
| 5 | 130 | 101.25 | 1.28 | 1.18 | 110.20 | 100.71 | 118.81 | 11.19 | 125.28 | 11.19 | 8.61 |
| 6 | 95 | 105.00 | 0.90 | 0.87 | 109.08 | 111.94 | 97.49 | -2.49 | 6.18 | 2.49 | 2.62 |
| 7 | 75 | 106.25 | 0.71 | 0.68 | 110.52 | 113.28 | 76.87 | -1.87 | 3.51 | 1.87 | 2.50 |
| 8 | 150 | 108.75 | 1.38 | 1.27 | 118.03 | 115.43 | 146.69 | 3.31 | 10.95 | 3.31 | 2.21 |
| 9 | 135 | 112.50 | 1.20 | 1.18 | 114.44 | 118.38 | 139.65 | -4.65 | 21.66 | 4.65 | 3.45 |
| 10 | 105 | 113.75 | 0.92 | 0.87 | 120.57 | 118.30 | 103.03 | 1.97 | 3.89 | 1.97 | 1.88 |
| 11 | 85 | 116.25 | 0.73 | 0.68 | 125.26 | 120.89 | 82.04 | 2.96 | 8.77 | 2.96 | 3.48 |
| 12 | 155 | 118.75 | 1.31 | 1.27 | 121.97 | 124.43 | 158.13 | -3.13 | 9.78 | 3.13 | 2.02 |
| 13 | 145 | 120.00 | 1.21 | 1.18 | 122.91 | 125.39 | 147.92 | -2.92 | 8.55 | 2.92 | 2.02 |
| 14 | 110 | 122.50 | 0.90 | 0.87 | 126.31 | 126.67 | 110.31 | -0.31 | 0.10 | 0.31 | 0.29 |
| 15 | 85 | 123.75 | 0.69 | 0.68 | 125.26 | 127.41 | 86.46 | -1.46 | 2.13 | 1.46 | 1.72 |
| 16 | 160 | 123.75 | 1.29 | 1.27 | 125.90 | 126.28 | 160.48 | -0.48 | 0.23 | 0.48 | 0.30 |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  | Seasonal calculations |  |  | Seasonal  | Adjusted |  |  |  |  |
|  |  | Year 1 | Year 2 | Year 3 | Year 4 | Means | Means |  | MSE | MAE | MAPE |
|  | Q1 |  | 1.28 | 1.20 | 1.21 | 1.23 | 1.18 |  | 16.75 | 3.06 | 2.59 |
|  | Q2 |  | 0.90 | 0.92 | 0.90 | 0.91 | 0.87 |  | RMSE |  |  |
|  | Q3 |  | 0.71 | 0.73 | 0.69 | 0.71 | 0.68 |  | 4.09 |  |  |
|  | Q4 |  | 1.38 | 1.31 | 1.29 | 1.33 | 1.27 |  |  |  |  |
|  |  |  |  |  | **Overall** | **1.04** | **1** |  |  |  |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Obs | eries | MA(4) | Detrended Series | Seasonal Factor | Deseasonalized Series | Deseasonalized Forecasts | Series Forecasts |
| 1 | 115 |  |  | =H21 | =B2/E2 |  |  |
| 2 | 90 |  |  | =H22 | =B3/E3 |  |  |
| 3 | 65 |  |  | =H23 | =B4/E4 |  |  |
| 4 | 135 |  |  | =H24 | =B5/E5 | 97.1724824306238 |  |
| 5 | 130 | =AVERAGE(B2:B5) | =B6/C6 | =H21 | =B6/E6 | 103.760077010048 | =G6\*E6 |
| 6 | 95 | =AVERAGE(B3:B6) | =B7/C7 | =H22 | =B7/E7 | 110.795468765077 | =G7\*E7 |
| 7 | 75 | =AVERAGE(B4:B7) | =B8/C8 | =H23 | =B8/E8 | 112.98651192633 | =G8\*E8 |
| 8 | 150 | =AVERAGE(B5:B8) | =B9/C9 | =H24 | =B9/E9 | 114.068665475918 | =G9\*E9 |
| 9 | 135 | =AVERAGE(B6:B9) | =B10/C10 | =H21 | =B10/E10 | 119.722768005783 | =G10\*E10 |
| 10 | 105 | =AVERAGE(B7:B10) | =B11/C11 | =H22 | =B11/E11 | 118.919548314113 | =G11\*E11 |
| 11 | 85 | =AVERAGE(B8:B11) | =B12/C12 | =H23 | =B12/E12 | 122.290614324503 | =G12\*E12 |
| 12 | 155 | =AVERAGE(B9:B12) | =B13/C13 | =H24 | =B13/E13 | 127.218682958443 | =G13\*E13 |
| 13 | 145 | =AVERAGE(B10:B13) | =B14/C14 | =H21 | =B14/E14 | 126.252087646846 | =G14\*E14 |
| 14 | 110 | =AVERAGE(B11:B14) | =B15/C15 | =H22 | =B15/E15 | 125.348712558888 | =G15\*E15 |
| 15 | 85 | =AVERAGE(B12:B15) | =B16/C16 | =H23 | =B16/E16 | 127.014755589409 | =G16\*E16 |
| 16 | 160 | =AVERAGE(B13:B16) | =B17/C17 | =H24 | =B17/E17 | 126.726947008494 | =G17\*E17 |

### 4.10

Equation (4.9) is 

Substituting into the level equation immediately gives



Results for the Trend and Seasonal state equations follow in the same way.

### 4.11

For the multiplicative scheme



Substituting into the level equation gives



Results for the Trend and Seasonal state equations follow in the same way.

### 4.12

There are 92 observed error terms so an approximate 95 percent interval is obtained by taking averages of the second and third smallest, and the second and third largest. For 2015Q1, we have the values -9.42 and +10.93 so the interval is

(118.7-9.42, 118.7+10.93) = (109.3, 129.6)

The normal interval was (106.9, 130.6), which is slightly wider.

One-step-ahead intervals could similarly be constructed for the other quarters of 2015, but to get 2 or more step-ahead intervals we would need to compute the 2, 3, … step-ahead errors; these are not provided by the ESM

### Minicases

### Minicase 4.1: Walmart Sales

Discuss the various methods; all are much better than the non-seasonal versions.

### Minicase 4.2: Automobile Production

Examine different estimation samples and their effect upon the prediction intervals, which are sensitive to such decisions, even when the point forecasts and the parameter estimates may not be.

### Minicase 4.3: U.S. Retail Sales

The basis ideas are similar to those in Minicase 4.2. It is important to understand the data!

### Minicase 4.4: UK Retail Sales

The analysis follows along the same lines as Exercise 4.6. Comparisons between the UK and the U.S. may relate to the structure of the series, parameter estimates and to forecast quality.

### Minicase 4.5: Newspaper Sales

Sales are rather erratic, but the example shows a seasonal analysis not related to weeks. Zero returns were assumed for the missing observation.

|  |
| --- |
| **Forecasting Information for Additive Seasonal Exponential smoothing** |
|   |   |
| Total Observations | 42 |
| Holdout | 0 |
| Forecast Horizon | 7 |
| Seasonal Cycle | 7 |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 2.406879 |
| Mean of Squared Error (MSE) | 9.296453 |
| Root Mean of Squared Error (RMSE) | 3.049008 |
| Mean of Absolute Percent Error (MAPE) | 0.293107 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.248127 |
|   |   |
| **Parameter** |   |
| alpha | 0.046776 |
| gamma | 0.268420 |
|   |   |