# Chapter 5: Outline Solutions

The macro file ES Macro\_Chapter 5 contains all the data sets used in Exercises for Chapter 4. The macro is being updated so the layout may change but the results should not. Users are encouraged to run the macro to gain insight into methods’ performance. ONLY out-of-sample results are recorded here, in most cases.

Note that the output includes many more decimal points than should be used when presenting results. Excel should be used to format them for presentational purposes.

### 5.1

For h-steps ahead, the point forecast is 

RMSE = 64，Substituting the values given in the formula,the RMSE's 1 through 4 steps ahead are: 64, 64\*sqrt(1.09)=66.82, 64\*sqrt(1.18)=69.52 and 64\*sqrt(1.27)=72.12.

The 95% prediction intervals are then 100 ± 1.96\*RMSE.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| h | Forecast | Forecast RMSE | Lower Limit | Upper Limit |
| A | B | D | E=B-Z\*D | E= B+ Z\*D |
| 1 | 100 | 64 | -25.44 | 225.44 |
| 2 | 100 | 66.82 | -30.96 | 230.96 |
| 3 | 100 | 69.52 | -36.27 | 236.26 |
| 4 | 100 | 72.12 | -41.36 | 241.36 |

### 5.2

*Refer back to Exercises 3.1 and 3.3*

 The Great recession has the effect of increasing the estimated in-sample RMSE and producing a larger value of α, so the prediction intervals, period by period are wider for the later data.

*Use data through 2007*

|  |
| --- |
| **Forecasting Information for Single Exponential smoothing** |
|   |   |
| Total Observations | 53 |
| Holdout | 8 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.592943 |
| Mean of Squared Error (MSE) | 4.124055 |
| Root Mean of Squared Error (RMSE) | 2.030777 |
| Mean of Absolute Percent Error (MAPE) | 0.258049 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.222961 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.864276 |
| Mean of Squared Error (MSE) | 7.082736 |
| Root Mean of Squared Error (RMSE) | 2.661341 |
| Mean of Absolute Percent Error (MAPE) | 0.061902 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 1.087427 |
|   |   |
| **Parameter** |   |
| alpha | 0.367171 |

*Using data through 2009*

|  |
| --- |
| **Forecasting Information for Single Exponential smoothing** |
|   |   |
| Total Observations | 53 |
| Holdout | 6 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.755399 |
| Mean of Squared Error (MSE) | 4.965600 |
| Root Mean of Squared Error (RMSE) | 2.228363 |
| Mean of Absolute Percent Error (MAPE) | 0.236543 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.393049 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 1.070758 |
| Mean of Squared Error (MSE) | 1.980352 |
| Root Mean of Squared Error (RMSE) | 1.407250 |
| Mean of Absolute Percent Error (MAPE) | 0.277762 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.375756 |
|   |   |
| **Parameter** |   |
| alpha | 0.466951 |
|   |   |

*The ESM macro provides the point forecasts and the limits*

|  |  |  |  |
| --- | --- | --- | --- |
| **2007 origin** |  |  |  |
|  | **Forecast** | **Lower Limit** | **Upper Limit** |
| 2008 | 3.676 | -0.350 | 7.702 |
| 2009 | 3.676 | -0.613 | 7.965 |
| 2010 | 3.676 | -0.860 | 8.213 |
| 2011 | 3.676 | -1.095 | 8.448 |
| 2012 | 3.676 | -1.319 | 8.671 |
| 2013 | 3.676 | -1.533 | 8.886 |
|  |  |  |  |
| **2009 origin** |  |  |  |
|  | **Forecast** | **Lower Limit** | **Upper Limit** |
| 2010 | 3.739 | -0.657 | 8.135 |
| 2011 | 3.739 | -1.112 | 8.591 |
| 2012 | 3.739 | -1.529 | 9.008 |
| 2013 | 3.739 | -1.915 | 9.393 |
| 2014 | 3.739 | -2.276 | 9.754 |
| 2015 | 3.739 | -2.616 | 10.095 |

### 5.3

*Refer back to Exercises 3.2 and 3.5*

The results for the two estimation samples are very similar. Although the PIs are wider for the shorter data set , the forecasts point to continued levels of inflation below the stated (Federal Reserve) target of 2%.

*Using data though 2012*

|  |
| --- |
| **Forecasting Information for Single Exponential smoothing** |
|   |   |
| Total Observations | 144 |
| Holdout | 0 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 0.466808 |
| Mean of Squared Error (MSE) | 0.347980 |
| Root Mean of Squared Error (RMSE) | 0.589898 |
| Mean of Absolute Percent Error (MAPE) | 0.295386 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.244725 |
|   |   |
| **Parameter** |   |
| alpha | 0.368763 |
|   |   |
|  |  |

*Using data through 2014*

|  |
| --- |
| **Forecasting Information for Single Exponential smoothing** |
|   |   |
| Total Observations | 168 |
| Holdout | 0 |
| Forecast Horizon | 12 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 0.449740 |
| Mean of Squared Error (MSE) | 0.327586 |
| Root Mean of Squared Error (RMSE) | 0.572351 |
| Mean of Absolute Percent Error (MAPE) | 0.308324 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.243762 |
|   |   |
| **Parameter** |   |
| alpha | 0.338773 |
|   |   |

*The ESM macro provides the point forecasts and the limits*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **2013** | **Forecast** | **Lower Limit** | **Upper Limit** |  | **2015** | **Forecast** | **Lower Limit** | **Upper Limit** |
| Jan | 1.324 | 0.166 | 2.482 |  | Jan | 1.447 | 0.324 | 2.570 |
| Feb | 1.324 | 0.090 | 2.558 |  | Feb | 1.447 | 0.261 | 2.633 |
| Mar | 1.324 | 0.018 | 2.630 |  | Mar | 1.447 | 0.201 | 2.693 |
| Apr | 1.324 | -0.050 | 2.698 |  | Apr | 1.447 | 0.145 | 2.750 |
| May | 1.324 | -0.115 | 2.763 |  | May | 1.447 | 0.090 | 2.804 |
| Jun | 1.324 | -0.177 | 2.825 |  | Jun | 1.447 | 0.038 | 2.856 |
| Jul | 1.324 | -0.236 | 2.884 |  | Jul | 1.447 | -0.013 | 2.907 |
| Aug | 1.324 | -0.294 | 2.942 |  | Aug | 1.447 | -0.061 | 2.956 |
| Sep | 1.324 | -0.349 | 2.997 |  | Sep | 1.447 | -0.109 | 3.003 |
| Oct | 1.324 | -0.403 | 3.051 |  | Oct | 1.447 | -0.155 | 3.049 |
| Nov | 1.324 | -0.455 | 3.103 |  | Nov | 1.447 | -0.199 | 3.093 |
| Dec | 1.324 | -0.505 | 3.153 |  | Dec | 1.447 | -0.243 | 3.137 |

### 5.4

From the observation equation, we have:



This follows because the best forecast of a future error is its expectation, which is zero. Likewise, the updated state equations become:

.

For two steps ahead, we must substitute the forecasts for the state equations:



Repeating the process, we arrive at:

.

Using the properties of the geometric series the limit as *h* goes to infinity is:



### 5.5

The point forecasts are given by . The FMSE uses the expression in the text:



Substituting for *h=*1,…,4 we obtain:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *h* | 1 | 2 | 3 | 4 |
|  |  |  |  |  |
| *V(h)* | 6.25 | 7.81 | 10.88 | 15.94 |
| *SE(h)* | 2.5 | 2.80 | 3.30 | 3.99 |

The prediction intervals may then be calculated in the usual way.

### 5.6

The (A,A,N) model offers no improvement over the (A,N,N) version. Further, the SES scheme is essentially a random walk with coefficient very close to 1.0.

Both models have prediction limits of +/- 8.3. In each case, 2 out of 36 observations lie outside the limits, close to 95 percent inside. However, it should be noted that the distribution appears to have long tails, so extreme values occur more often than would happen with the normal distribution.

|  |  |
| --- | --- |
|  |  |
| **Forecasting Information for Single Exponential smoothing** |
|   |   |
| Total Observations | 312 |
| Holdout | 36 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 2.877496 |
| Mean of Squared Error (MSE) | 17.922385 |
| Root Mean of Squared Error (RMSE) | 4.233484 |
| Mean of Absolute Percent Error (MAPE) | 0.132685 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.132375 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 3.135014 |
| Mean of Squared Error (MSE) | 19.841074 |
| Root Mean of Squared Error (RMSE) | 4.454332 |
| Mean of Absolute Percent Error (MAPE) | 0.193261 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.190263 |
|   |   |
| **Parameter** |   |
| alpha | 0.992385 |
|   |   |
|   |   |

|  |
| --- |
| **Forecasting Information for Additive Trend Exponential smoothing** |
|   |   |
| Total Observations | 312 |
| Holdout | 36 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 2.997079 |
| Mean of Squared Error (MSE) | 18.974045 |
| Root Mean of Squared Error (RMSE) | 4.355921 |
| Mean of Absolute Percent Error (MAPE) | 0.138186 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.140656 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 3.203761 |
| Mean of Squared Error (MSE) | 20.564199 |
| Root Mean of Squared Error (RMSE) | 4.534777 |
| Mean of Absolute Percent Error (MAPE) | 0.196871 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.195075 |
|   |   |
| **Parameter** |   |
| alpha | 1.000000 |
| beta | 0.029868 |
|   |   |

### 5.7

The one-step-ahead prediction interval for the logarithm is:

 

Converting back to the original units (i.e. using the EXP function) yields the prediction interval. OK, the numbers are huge!!

### 5.8

Using the ESM, the transform to logs makes little difference to the analysis of the VIX data. Only the output for the (A,N,N) model is shown.

The estimated standard deviation for the Log-errors is 0.173, so the 95 percent prediction intervals are +/- 0.339. The log model is less effective over the hold-out period as 5 out of 36 observations lie outside the range.

|  |
| --- |
| **Forecasting Information for Single Exponential smoothing** |
|   |   |
| Total Observations | 312 |
| Holdout | 36 |
| Forecast Horizon | 10 |
|   |   |
|   |   |
| **In sample Error** |   |
| Mean of Absolute Deviation (MAD) | 2.880364 |
| Mean of Squared Error (MSE) | 17.923043 |
| Root Mean of Squared Error (RMSE) | 4.233561 |
| Mean of Absolute Percent Error (MAPE) | 0.132864 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.132538 |
|   |   |
| **Out of sample Error** |   |
| Mean of Absolute Deviation (MAD) | 3.148056 |
| Mean of Squared Error (MSE) | 19.962292 |
| Root Mean of Squared Error (RMSE) | 4.467918 |
| Mean of Absolute Percent Error (MAPE) | 0.194047 |
| Symmetric Mean of Absolute Percent Error (sMAPE) | 0.191046 |
|   |   |
| **Parameter** |   |
| alpha | 1.000000 |
|   |   |

### 5.9

The following models are listed below (multiplicative seasonals did not perform well) were considered:

Local level

Local trend

Local level + additive seasonal

Local Trend + additive seasonal

Damped trend + additive seasonal

Local trend + additive seasonal + Box Cox transform.

The Box-Cox transform produced an optimal value of 1.0 (= no transform) and is not considered further).

The out-of-sample criteria favor “Local level + additive seasonal” whereas the information criteria all favor “Damped trend + additive seasonal”.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Model | (A,N,N) | (A,A,N) | (A,N,A) | (A,A,A) | (A,D,A) |
| In-sample |  |  |  |  |  |
| MAE | 5.68 | 5.51 | 1.91 | 1.61 | 1.55 |
| MSE | 58.81 | 41.32 | 6.49 | 4.48 | 3.99 |
| RMSE | 7.67 | 6.43 | 2.62 | 2.12 | 2.00 |
| MAPE | 5.90 | 5.92 | 2.11 | 1.78 | 1.70 |
| Hold-out  |  |  |  |  |  |
| MAE | 5.61 | 6.67 | 1.19 | 1.45 | 1.32 |
| RMSE | 7.31 | 7.14 | 1.51 | 1.80 | 1.67 |
| MAPE | 4.60 | 5.62 | 1.01 | 1.22 | 1.11 |
| α | 0.45 | 0.04 | 1.00 | 0.67 | 0.59 |
| β |  | 1.00 |  | 0.07 | 0.00 |
| γ |  |  | 0.20 | 0.23 | 0.26 |
| ϕ |  |  |  |  | 0.98 |

Information Criteria

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Model | ANN | AAN | ANA | AAA | ADA |
| n | 52 | 52 | 52 | 52 | 52 |
| p | 1 | 2 | 2 | 3 | 4 |
| MSE | 58.81 | 41.32 | 6.49 | 4.475 | 3.99 |
| AIC | 4.113 | 3.798 | 1.947 | 1.614 | 1.538 |
| BIC | 4.150 | 3.873 | 2.022 | 1.726 | 1.688 |
| AICc | 4.114 | 3.803 | 1.952 | 1.624 | 1.554 |
| Hold-out |  |  | \*\*\* |  |  |

In Exercises 5.10-5.14 the original estimation sample was used to calculate the values of the information criteria, to reduce the amount of computation required. The correct calculation should use the complete data set. Readers with sufficient stamina are encouraged to re-run the analyses with the full series.

### 5.10

The effects of the recession lead to the “no seasonal” choice for the information criteria. The seasonal pattern reappears in the out-of-sample data.

|  |  |  |  |
| --- | --- | --- | --- |
| **Job Opportunities** |  |  |  |
| Model | ANN | ANA | ANM | AAA |
| n | 144 | 144 | 144 | 144 |
| p | 1 | 2 | 2 | 3 |
| MSE | 46163 | 61963 | 63130 | 61445 |
| AIC | 10.754 | 11.062 | 11.081 | 11.068 |
| BIC | 10.774 | 11.103 | 11.122 | 11.129 |
| AICc | 10.754 | 11.063 | 11.081 | 11.069 |
| Hold-out |  |  |  | \*\*\* |

### 5.11

The trend + additive seasonals model is the clear choice.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Alcohol Sales** |  |  |  |  |
| Model | ANN | ANA | AAA | AAM | MMM |
| n | 144 | 144 | 144 | 144 | 144 |
| p | 1 | 2 | 3 | 3 | 3 |
| MSE | 1045121 | 163616 | 145916 | 154753 | 159316 |
| AIC | 13.874 | 12.033 | 11.932 | 11.991 | 12.020 |
| BIC | 13.894 | 12.074 | 11.994 | 12.053 | 12.082 |
| AICc | 13.874 | 12.034 | 11.934 | 11.992 | 12.022 |
| Hold-out |  |  | \*\*\* |  |  |

### 5.12

The seasonal models are all very close, so the different choices have little impact on forecasting.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Retail Sales** |  |  |  |  |  |
| Model | ANN | ANA | AAA | AAM | MMM |
| n | 144 | 144 | 144 | 144 | 144 |
| p | 1 | 2 | 3 | 3 | 3 |
| MSE | 608.62 | 50.45 | 49.08 | 49.13 | 49.21 |
| AIC | 6.425 | 3.949 | 3.935 | 3.936 | 3.938 |
| BIC | 6.446 | 3.990 | 3.997 | 3.998 | 4.000 |
| AICc | 6.425 | 3.949 | 3.936 | 3.937 | 3.939 |
| Hold-out |  |  |  |  | \*\*\* |

### 5.13

The seasonal model with no trend term is selected. The reducing amplitude of the series clearly affects the choice for this unusual series.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Titanic** |  |  |  |  |  |
| Model | ANN | ANA | AAA | AAM | MMM |
| n | 158 | 158 | 158 | 158 | 158 |
| p | 1 | 2 | 3 | 3 | 3 |
| MSE | 1000533 | 182725 | 207318 | 205809 | 235030 |
| AIC | 13.829 | 12.141 | 12.280 | 12.273 | 12.405 |
| BIC | 13.848 | 12.180 | 12.338 | 12.331 | 12.464 |
| AICc | 13.829 | 12.142 | 12.281 | 12.274 | 12.406 |
| Hold-out |  | \*\*\* |  |  |  |

### 5.14

The penalty is sufficient to move the choice from a 3-parameter to a 2-parameter model. The differences are slight.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Gas Prices** |  |  |  |  |  |  |
| Model | ANN | AAN | ANA | AAA | AAM | MMM |
| n | 156 | 156 | 156 | 156 | 156 | 156 |
| p | 1 | 2 | 2 | 3 | 3 | 3 |
| MSE | 305.47 | 312.3 | 274.77 | 276.08 | 273.07 | 275.41 |
| AIC | 5.735 | 5.770 | 5.642 | 5.659 | 5.648 | 5.657 |
| BIC | 5.754 | 5.809 | 5.681 | 5.718 | 5.707 | 5.715 |
| AICc | 5.735 | 5.770 | 5.642 | 5.660 | 5.649 | 5.658 |
| Hold-out |  |  |  |  | \*\*\* |  |

### Minicase 5.1

Incorporate seasonal effects in the quarterly series and allow for trend in both.

### Minicase 5.2

Note how the series moves to a new level after time 12, leading to narrower intervals. What should be done about the Christmas season?